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EFFECT OF CONCRETE-REPRESENTATIONAL-ABSTRACT AS TEACHING APPROACH ON LEARNERS' NUMERACY SKILL

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Abstract

This study aimed to determine the effectiveness of the Concrete–Representational–Abstract (CRA) approach in enhancing the numeracy skills of Grade 11 TVL (Electrical Installation and Maintenance) learners at Major Manuel A. Aaron Memorial National High School for the school year 2024–2025. Utilizing a quasi-experimental design with a non-equivalent control group, the study involved two groups: the experimental group, which received instruction using the CRA method, and the control group, which was taught using traditional methods. Both groups underwent a pre-test and post-test using the researcher-made questionnaire to assess their numeracy performance. The pre-test results showed that both the experimental and control groups were at a Moderate level of performance, indicating comparable baseline numeracy skills. After the intervention, the experimental group's performance improved to a high level, while the control group remained in the Moderate range. A paired sample t-test revealed a statistically significant difference in the experimental group's pre-test and post-test scores ($t = -2.863$, $p = .010$), while the control group showed no significant improvement ($t = 1.261$, $p = .223$). Moreover, an independent t-test confirmed a significant difference in post-test performance between the two groups, favoring the experimental group. These findings suggest that the CRA approach significantly enhances learners' numeracy skills by providing a scaffolded learning structure that transitions from concrete manipulatives to abstract symbols. The study concludes that the CRA method is more effective than traditional instruction in developing mathematical understanding among senior high school learners. Integrating CRA into mathematics instruction is therefore recommended to address persistent numeracy gaps and promote higher-level mathematical thinking in technical-vocational education contexts.

Keywords: *Concrete, Representational, Abstract, Math Instruction*

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1. Introduction

Mathematics education plays a central role in equipping learners with the skills needed to solve real-world problems, and numeracy in particular is reinforced across grade levels through varied intervention and enrichment programs. A key component of this instruction is the problem-solving approach, which requires students to apply, synthesize, and extend prior knowledge to non-routine tasks. Both teachers and students, however, continue to struggle with this process — students in approaching and resolving complex problems, and teachers in delivering instruction that builds problem-solving proficiency. Evidence suggests that learners who engage with concrete materials develop stronger mental representations, greater motivation, and deeper conceptual understanding, benefits that extend across topics such as place value, computation, fractions, measurement, and word problems.

One instructional model built on this premise is the Concrete-Representational-Abstract (CRA) approach, a three-stage strategy in which learners progress from manipulating physical objects (concrete), to using drawings or diagrams (representational), to working with mathematical symbols (abstract). Originally developed for learners with mathematical difficulties, CRA has since shown benefits for the general student population as well. Despite these documented benefits and continuing efforts to raise mathematics achievement, many Philippine senior high school learners still register low scores on researcher-made and competency-based numeracy assessments. This gap motivated the present study, which examined the effect of the CRA instructional approach on the numeracy skills of Grade 11 TVL (Electrical Installation and Maintenance) learners at Major Manuel A. Aaron Memorial National High School, Janiuay, Iloilo, for School Year 2024–2025.

1.1 Research Questions and Hypotheses

Specifically, the study sought to answer: (1) What was the level of learners' numeracy skill in the experimental and control groups before the intervention? (2) What was it after the intervention? (3) Were there significant differences between groups before the intervention? (4) After the intervention? (5) Within each group, before and after the intervention? (6) Was there a significant difference in mean gain between groups? For each question, the corresponding null hypothesis was tested at the 0.05 level of significance: that no significant difference existed between or within groups on the measures described above.

1.2 Theoretical Framework

The study drew on Cognitive Apprenticeship Theory (Medical College of Wisconsin, 2022) and Piaget's Theory of Cognitive Development. Cognitive Apprenticeship Theory emphasizes learning within authentic contexts where students observe, practice, and refine skills under the guidance of a mentor — a framework that aligns with CRA's emphasis on guided participation and progressive mastery. The CRA sequence is itself grounded in constructivist learning theory, in which learners build deep understanding by moving through hands-on, visual, and symbolic stages. Piaget's theory complements this by explaining why: children in the concrete operational stage (roughly ages 7–11) reason logically only within tangible contexts, and the transition toward formal operational, abstract reasoning is gradual. Instructional strategies like CRA are therefore positioned to bridge this cognitive gap, consistent with the Department of Education's broader mathematics curriculum goals of building both numeracy skills and positive attitudes toward the subject.

1.3 Related Literature

International studies provide converging evidence for CRA's effectiveness. Goya (2023) found a clear positive relationship between CRA instruction and improved algebraic problem-solving among middle school students, with participants also reporting increased motivation. Flores, Hinton, and Meyer (2018/2020) compared CRA instruction to typical Tier 2 support within a Multi-Tiered System of Supports framework and found significantly greater improvement in fraction understanding among the CRA group. Sterner, Wolff, and Helenius (2019), in a longitudinal randomized study of Grade 0 learners, reported that CRA-based number sense gains were sustained at a nine-month follow-up. Prosser and Bismarck (2023) extended this work to secondary Algebra I, finding

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significant pre- to post-test gains, though students themselves did not always perceive the approach as beneficial — a finding the authors linked to a possible stigma around manipulative use at the secondary level.

Philippine studies echo these results. Reyes (2021) found that Grade 8 students taught probability using the CRA sequence significantly outperformed a traditionally-taught control group ($t = 7.71$). Gallo-Toong (2020), surveying 267 Grade 8 students across five schools in Negros Occidental, found CRA use widely reported across topics, though roughly a quarter of students remained at a "developing" performance level — suggesting that implementation quality, not just presence, determines CRA's impact. Morata et al. (2024) documented significant gains in kindergarten addition skills following CRA instruction, and Camañan (2019) similarly reported improved mathematics performance using the approach.

Taken together, the literature supports CRA as an effective, developmentally grounded strategy for building numeracy, but also suggests its benefits are not automatic — implementation quality, learner stigma, and instructional context all moderate outcomes. Few studies, however, have examined CRA specifically among senior high school Technical-Vocational-Livelihood learners, a population whose curricular track emphasizes applied and hands-on learning. This gap is the specific focus of the present study.

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2. Methods

2.1 Research Design

This study used a quantitative, two-group pretest-posttest quasi-experimental design to determine whether the CRA instructional approach, delivered as a mathematics enrichment intervention, produced a significant improvement in numeracy performance relative to conventional instruction.

2.2 Participants and Sampling

The researcher initially screened 50 students (42 male, 8 female) from two heterogeneous Grade 11 TVL-EIM classes using the DepEd Region VI researcher-made and competency-based questionnaire (mandated under Regional Memo No. 827, s. 2022) as a pretest. Using a pair-matching technique based on pretest scores, 40 students were identified and randomly assigned into two equal, comparable groups: an experimental group ($n = 20$) that received CRA-based instruction, and a control group ($n = 20$) that received conventional instruction. Pair-matching on baseline scores was intended to minimize confounding and ensure the two groups began the study with comparable numeracy levels.

2.3 Instrument

The primary instrument was the DepEd Region VI researcher-made and competency-based questionnaire, a standardized tool used to assess fundamental numeracy competencies — the four arithmetic operations and related problem-solving — for learners in Grades 1 through 11. The same instrument was administered at pretest (for screening and pair-matching) and posttest (to measure change). Scores were interpreted using a five-band scale: 0.00–10.00 Very Low, 10.01–20.00 Low, 20.01–30.00 Moderate, 30.01–40.00 High, and 40.01–50.00 Very High.

2.4 Validity and Reliability

As a DepEd Region VI-developed instrument, the questionnaire's content validity was established through expert panel review aligning items with DepEd learning competencies, and its reliability was reported by the source thesis to have been established through pilot testing and "calculating Cronbach's alpha and other reliability coefficients."

2.5 Procedure

After securing approval from the school principal, the researcher conducted an orientation with participants and parents to explain the study's purpose, procedures, and their roles, and obtained informed consent. Following pretesting and pair-matching, the eight-week intervention was delivered during Independent Classroom Learning (ICL) periods to avoid disrupting regular classes: the control group met from 1:00–2:00 PM and the experimental group from 3:00–4:00 PM, once weekly. The experimental group's CRA-based instruction focused on the four fundamental operations and used activity sheets, physical manipulatives, and multimedia presentations across the three CRA stages, with weekly formative assessments recorded on a monitoring tool shared with parents. The control group continued with conventional, teacher-centered instruction over the same period. At the end of the eight weeks, both groups completed the same instrument as a posttest.

2.6 Data Analysis

Descriptive statistics (frequency, mean, standard deviation) summarized group performance at each phase. Paired-samples t-tests assessed within-group pretest-to-posttest change for the experimental and control groups separately, and independent-samples t-tests compared the two groups at pretest and at posttest. All tests used a significance threshold of $p < .05$, consistent with the source thesis's statistical treatment (SPSS).

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2.7 Ethical Considerations

The study proceeded only after principal approval and informed consent from participants and parents. Participant responses were encoded using anonymous codes to protect confidentiality, and all data were used strictly for the purposes of this research.

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3. Results

3.1 Baseline (Pretest) Performance

Table 1 presents pretest performance. The control group's mean score was 24.40 (SD = 6.55) and the experimental group's was 24.90 (SD = 5.60); both fell within the Moderate performance band, as did the combined mean of 24.65 (SD = 6.02).

Table 1. Pretest Performance of Grade 11 TVL-EIM Learners

Group	n	Treatment	Mean	SD	Description
Control	20	Traditional Method	24.40	6.55	Moderate
Experimental	20	CRA	24.90	5.60	Moderate
Total	40	—	24.65	6.02	Moderate

Scale: 0.00–10.00 Very Low; 10.01–20.00 Low; 20.01–30.00 Moderate; 30.01–40.00 High; 40.01–50.00 Very High.

An independent-samples t-test found no significant difference between groups at pretest (Table 2), confirming baseline equivalence between the experimental and control groups before the intervention.

Table 2. Pretest Group Difference (Independent-Samples t-Test)

Variance Assumption	F	Sig.	t	df	p	Interpretation
Equal variances assumed	.777	.384	-.260	38	.797	Not Significant
Equal variances not assumed	—	—	-.260	38.106	.797	—

$p < .05$ is significant.

3.2 Posttest Performance

Table 3 presents posttest performance. The control group's mean rose to 26.40 (SD = 4.82), remaining Moderate, while the experimental group's mean rose to 30.20 (SD = 5.83), reaching the High band. The combined posttest mean was 26.30 (SD = 6.59).

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Table 3. Posttest Performance of Grade 11 TVL-EIM Learners

Group	n	Treatment	Mean	SD	Description
Control	20	Traditional Method	26.40	4.82	Moderate
Experimental	20	CRA	30.20	5.83	High
Total	40	—	26.30	6.59	Moderate

Scale: 0.00–10.00 Very Low; 10.01–20.00 Low; 20.01–30.00 Moderate; 30.01–40.00 High; 40.01–50.00 Very High.

The between-group difference at posttest was statistically significant (Table 4): the experimental group significantly outperformed the control group after the intervention.

Table 4. Posttest Group Difference (Independent-Samples t-Test)

Variance Assumption	F	Sig.	t	df	p	Interpretation
Equal variances assumed	1.165	.287	-4.614	38	.000	Significant
Equal variances not assumed	—	—	-4.614	36.700	.000	—

$p < .05$ is significant.

3.3 Within-Group Change

Table 5 shows the control group's pretest-to-posttest change (24.40 to 26.40); a paired-samples t-test (Table 6) found this change was not statistically significant.

Table 5. Control Group Performance Before and After the Intervention

Phase	Mean	SD	Description
Before	24.40	6.55	Moderate
After	26.40	4.82	Moderate

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Table 6. Control Group Paired-Samples t-Test

Group	t	df	p	Interpretation
Control	1.261	19	.223	Not Significant

p < .05 is significant.

Table 7 shows the experimental group's pretest-to-posttest change (24.90 to 30.20); a paired-samples t-test (Table 8) found this change was statistically significant.

Table 7. Experimental Group Performance Before and After the Intervention

Phase	Mean	SD	Description
Before	24.90	5.60	Moderate
After	30.20	5.83	High

Table 8. Experimental Group Paired-Samples t-Test

Group	t	df	p	Interpretation
Experimental	-2.863	19	.010	Significant

p < .05 is significant.

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4. Discussion

Baseline equivalence between the two groups (Table 1, Table 2) established a fair starting point for comparison: both groups entered the study with a Moderate and statistically indistinguishable level of numeracy — consistent with prior observations that technical-vocational learners often show a general but incomplete grasp of fundamental mathematics skills (Nguyen & Lee, 2021).

The experimental group's shift from Moderate to High performance, alongside a statistically significant within-group gain (Tables 7 and 8), indicates that the CRA sequence meaningfully improved learners' numeracy. This is consistent with Asuncion and Laurente (2021), who describe CRA as a cognitive bridge for learners struggling with abstract numerical reasoning, and with Dizon and Mercado (2021), who found that senior high school students exposed to CRA-based lessons showed greater retention and procedural fluency. The pattern also aligns with Bruner's enactive–iconic–symbolic progression as applied to modern pedagogy (Yu & Suh, 2019), and with Tieng and Fernandez (2020), who reported stronger engagement and fewer conceptual errors among CRA-taught learners on operations and problem-solving tasks — the same skill areas measured in this study.

By contrast, the control group's small, non-significant gain (Tables 5 and 6) suggests that conventional, teacher-centered instruction was less effective at deepening conceptual understanding during the study period. This mirrors Barrot (2019), who linked lecture-and-drill formats to limited engagement, and Dela Cruz and Almero (2020), who argued that traditional instruction often fails to meet the numeracy needs of diverse learners. Fajardo and Reyes (2021) and Chiu and Chan (2019) similarly found that innovative, scaffolded pedagogies such as CRA outperform traditional instruction in moving learners from moderate to high proficiency.

The significant between-group difference favoring the experimental group at posttest (Table 4) reinforces this pattern and is consistent with Muthusamy and Lim (2021), who found CRA-taught students significantly outperformed conventionally-taught peers on abstract problem application, and Lim and Tan (2020), who linked CRA's structured concrete-to-abstract bridge to stronger internalization of numerical relationships. Comparable Philippine findings (Wahyuni et al., 2022; Salinas & Concepcion, 2021) similarly report that multi-representational instruction outperforms conventional teaching on numeracy tasks.

Taken together, these results support the study's central premise: the CRA approach's structured, three-stage progression — grounded in Cognitive Apprenticeship Theory and Piaget's account of concrete-to-formal reasoning — appears well suited to Technical-Vocational-Livelihood learners, who typically benefit from hands-on, applied instruction. The gains observed here occurred over a relatively short eight-week enrichment period, suggesting the approach may be practical to adopt within existing DepEd enrichment structures without requiring a full curricular overhaul.

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5. Conclusion and Recommendations

5.1 Conclusions

Before the intervention, learners in both groups exhibited comparable Moderate-level numeracy proficiency, and the absence of a significant pretest difference confirms this baseline equivalence, strengthening the internal validity of subsequent comparisons. Following the intervention, learners taught using the CRA approach progressed from Moderate to High performance, a statistically significant within-group gain, while learners taught using the conventional method showed only a modest, non-significant improvement that remained within the Moderate range. At posttest, the CRA group significantly outperformed the conventionally-taught group. Together, these results indicate that the CRA instructional approach produced greater and statistically significant learning gains in numeracy than conventional instruction among Grade 11 TVL-EIM learners over the study period.

5.2 Limitations

This study was limited to a single strand (TVL-EIM) and grade level within one public high school, and its sample (N = 40) — while adequate for the chosen design — limits generalizability. The researcher-made and competency-based questionnaire was the sole assessment instrument used to measure numeracy gains, and the intervention was constrained by an eight-week implementation window and the logistical demands of after-hours scheduling.

5.3 Recommendations

1. Integrate the CRA approach into mathematics instruction, particularly within technical-vocational tracks, given its demonstrated effect on numeracy gains.
2. Provide continuous professional development — in-service training, seminars, or workshops — to help mathematics teachers design and facilitate CRA-based lessons consistently.
3. Develop and distribute instructional materials anchored on the CRA framework, including manipulatives, visual models, and abstract-stage exercises.
4. Pilot the CRA approach in other learning areas and grade levels to assess its broader applicability beyond senior high school numeracy.
5. Pursue further research on the long-term retention of CRA-related gains, its integration with digital tools, and its effectiveness with learners with special educational needs.

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